Dr. Norbert Cheung's Series in Electrical Engineering

Level 2 Topic no: 08

Analogue Computer Simulation

1 diff equation → simulation circuit
2 circuit → diff equation

Contents

1. Elements of an analogue computer
2. Deriving the analogue simulation circuit
3. Deriving the equation from an analogue computer circuit
4. Deriving the circuit from the S-domain transfer function

Reference:
Chapter 8, G. Rissoni, "Principles and Applications of Electrical Engineering," 4th Edition, McGraw Hill.

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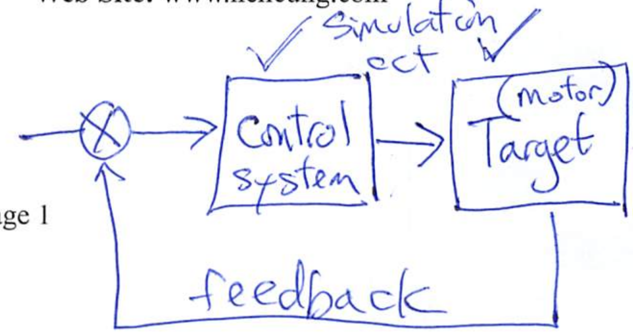
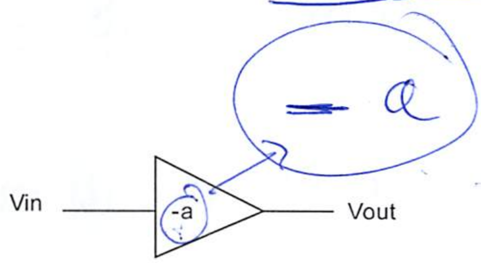


1. Elements of an analogue computer

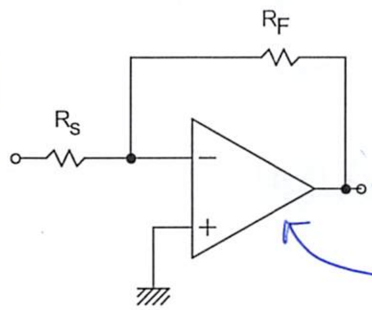
Operational amplifiers make up the basic functional block of analogue computers. Op-amp must have the following criteria:

1. Very low offset error and very low drift.
2. High differential gain and very high input impedance.
3. All capacitors must be fully discharged before the simulation

Inverting Amplifier scaling function



Symbol Representation



Actual Circuit

Negative function

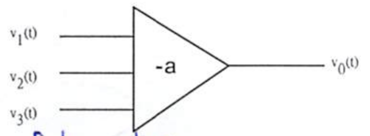
Transfer function = $-a = -R_F/R_S$

Inverting amplifier functions as voltage scalar or inversion in an analogue computer. Non-inverting amplifier is seldom used in an analogue computer.

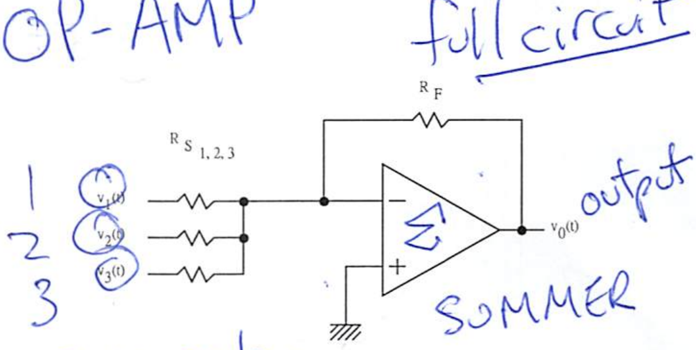
Summing Amplifier

OP-AMP

full circuit



block diagram form



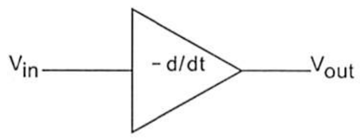
gain scale

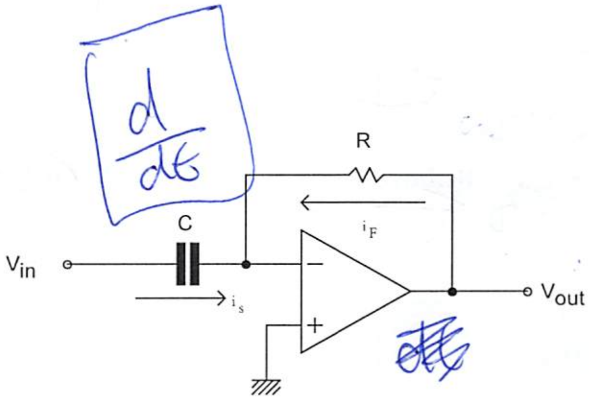
Output Function: $V_O = -\frac{R_F}{R_{S1}}V_1 + \frac{R_F}{R_{S2}}V_2 + \frac{R_F}{R_{S3}}V_3$

The circuit functions as signal summer in an analogue computer. Non-inverting summer is seldom used.

Differentiator

not often used





$i_s = C\frac{dV_{in}}{dt}$ and $i_F = \frac{V_{out}}{R}$ and $i_s + i_F = 0$

make it 1

$R = 1M\Omega$
$C = 1\mu F$
$R \times C = 1$

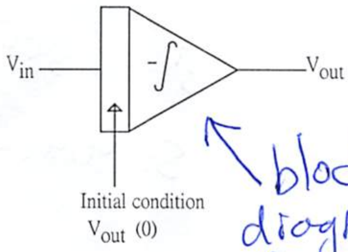
$\boxed{V_{out} = -RC\frac{dV_{in}}{dt}}$ ← diff function

In practice, the differential block is seldom used in analogue computer, because of the problem of "differential noise".

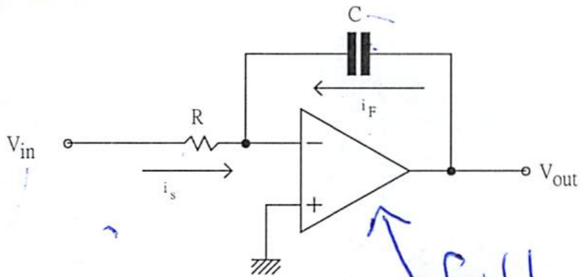
negative

Integrator

$\int dt$



block diagram form



full circuit form

$$i_s = \frac{V_{in}}{R} \quad \text{and} \quad i_F = C\frac{dV_{out}}{dt} \quad \text{and} \quad i_s + i_F = 0$$

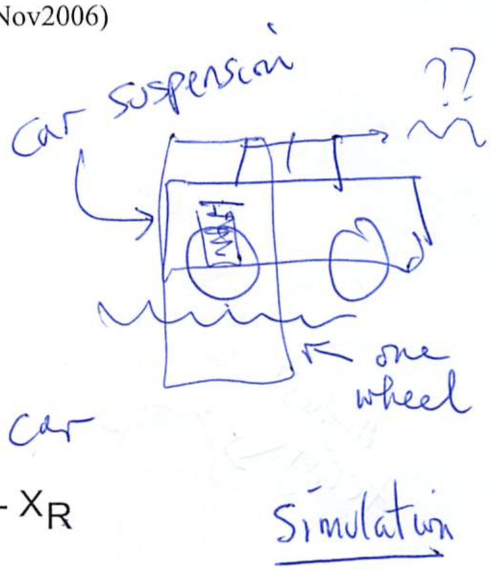
$$\frac{1}{RC}V_{in} = -\frac{dV_{out}}{dt} \qquad \text{therefore:} \qquad \boxed{V_{out} = \int V_{in}\,dt}$$

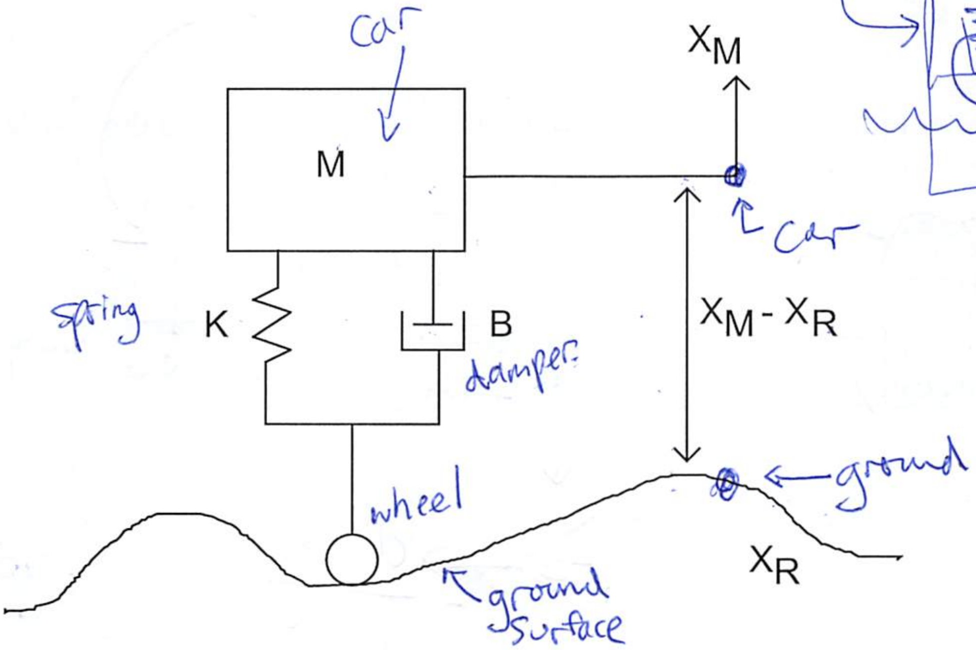
$R \times C = 1$

C

In most cases, the integration will be discharged fully before simulation takes place. Therefore the integration duration starts from *t=0*.

2. Deriving the analogue simulation circuit





M: Mass K: Spring Constant B: Damping Coefficient
X_M: Position of Mass X_R: Position of ground

The dynamic equation is a typical second order motion mechanics

$$\underbrace{M\frac{d^2x_M}{dt^2}}_{1}+\underbrace{B\left(\frac{dx_M}{dt}-\frac{dx_R}{dt}\right)}_{2}+\underbrace{K(x_M-x_R)}_{3}=0$$

1 Mass → F = ma
2 Damper → F = B v
3 Spring → F = K x

$$\boxed{M\frac{d^2x_M}{dt^2}+B\frac{dx_M}{dt}+Kx_M=B\frac{dx_R}{dt}+Kx_R}$$

Assume the road profile is a sinusoidal curve

$$\left.\begin{aligned} x_R &= X\sin(\omega t) \\ \frac{dx_R}{dt} &= \omega X\cos(\omega t) \end{aligned}\right\} \text{Road}$$

We can then write the equation in the form:

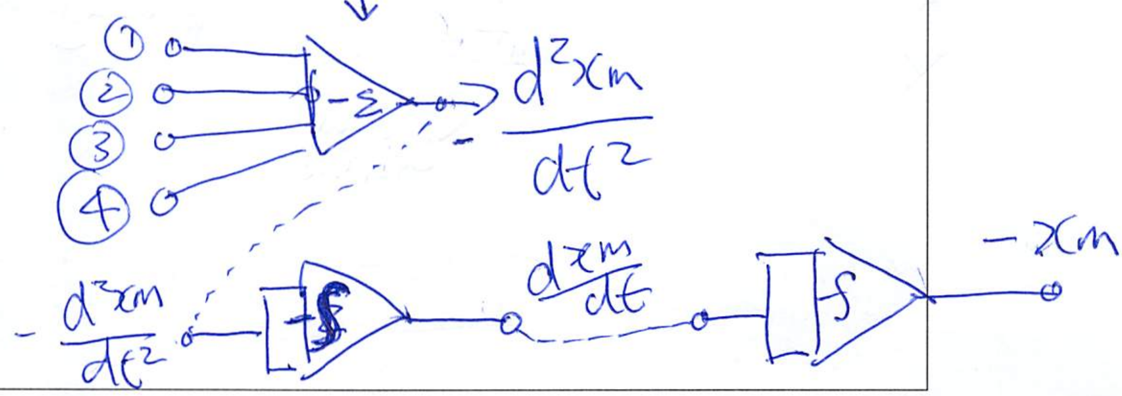
$$\underbrace{\boxed{M\frac{d^2x_M}{dt^2}+B\frac{dx_M}{dt}+Kx_M}}_{\text{3 forces}}=\underbrace{\boxed{B\omega X\cos(\omega t)+KX\sin(\omega t)}}_{\text{force due to road surface}}$$

Rearrange the highest order term to the left hand side, and the rest to the right hand side.

highest term

$$\boxed{\frac{d^2x_M}{dt^2}=\underbrace{-\overset{1}{\frac{B}{M}\frac{dx_M}{dt}}-\overset{2}{\frac{K}{M}x_M}+\overset{3}{\frac{B}{M}\omega X\cos(\omega t)}+\overset{4}{\frac{K}{M}X\sin(\omega t)}}_{\text{everything else.}}}$$

The analogue simulation diagram will be:



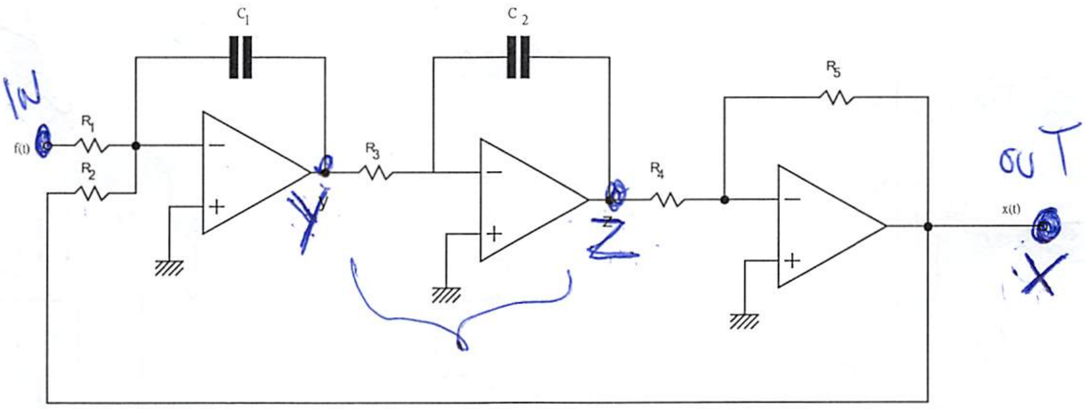
The full circuit diagram will be

3. Deriving the equation from an analogue simulation circuit

2nd technique

For the circuit below, obtain its transfer function

$R_1=0.4M\Omega$, $R_2=R_3=R_5=1M\Omega$, $R_4=2.5k\Omega$, $C_1=C_2=1\,\mu F$



STEP 1: Mark the important signal points

STEP 2: Define the relations of each block

x =

z =

y =

STEP 3: Substitute and eliminating the intermediate variable

DONE!

3. Deriving the circuit from the S-domain transfer function

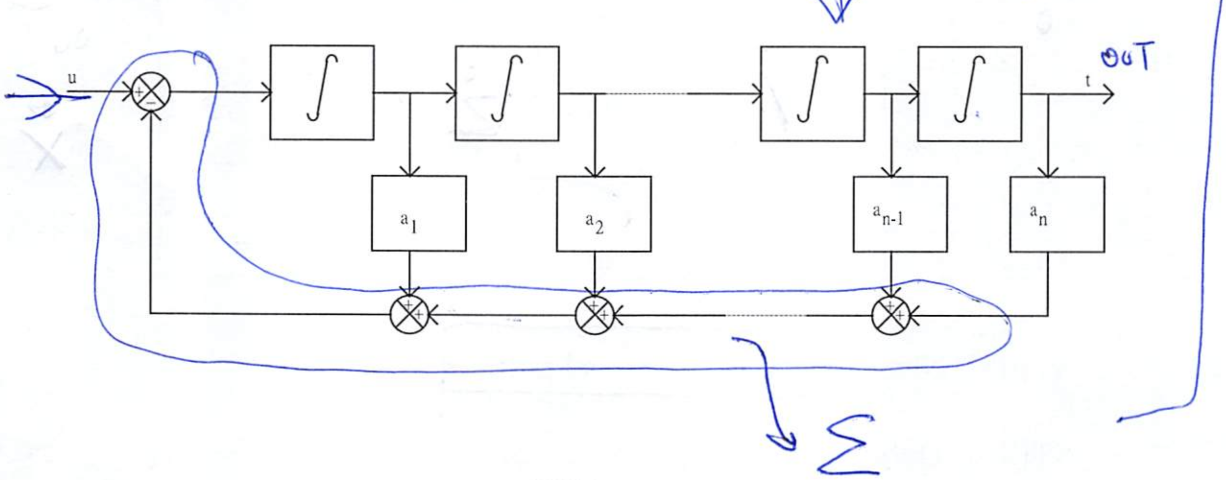
In S-domain, for the general transfer function of:

theoretic control

T.F.:
$$\frac{Y(s)}{U(s)} = \frac{1}{S^n + a_1 S^{n-1} + \square + a_{n-1} s + a_n}$$

. . . . .

The circuit can be constructed as:



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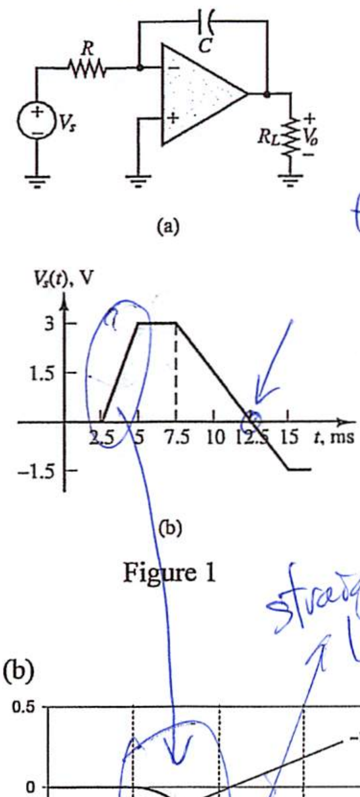
Tutorial Problems 1: Analog Computer Simulation

1. The circuit shown in Figure 1(a) will give an output voltage which is the integral of the supply voltage shown in Figure 1(b) multiplied by some gain.

   Determine:
   a. An expression for the output voltage.
   b. The value of the output voltage at $t = 5$, 7.5, 12.5, 15, and 20 ms and a plot of the output voltage as a function of time if

   $C = 1\ \mu F$ $\qquad R = 10\ k\Omega$ $\qquad R_L = 1\ k\Omega$

$R \times C = \frac{1}{100}$



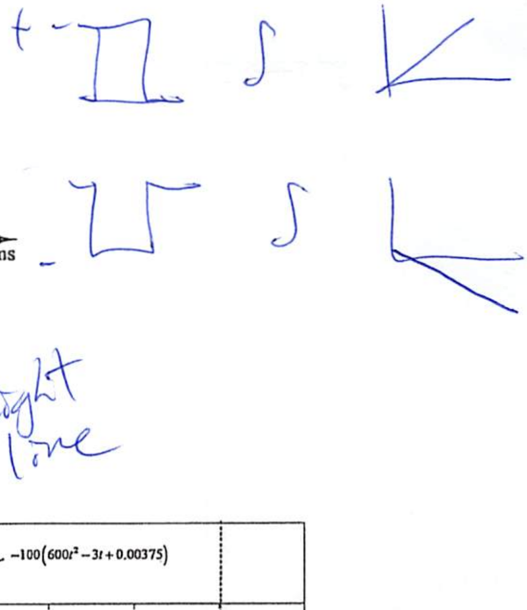


Figure 1

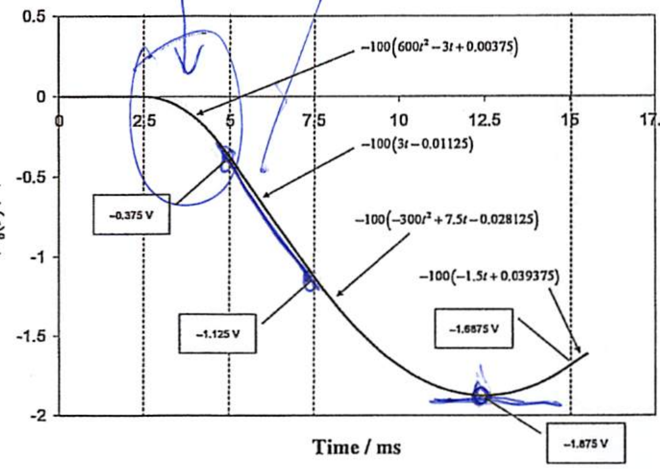
straight line

Solution:

(a)

$$V_o(t) = -\frac{1}{RC}\int V_S(t)\,dt$$
$$= -100\int V_S(t)\,dt$$

(b)



2. Derive the differential equation corresponding to the analog computer simulation circuit of Figure 2.

$y \leftrightarrow z$ $\quad$ $z \leftrightarrow x$

$f(t) \leftrightarrow y$

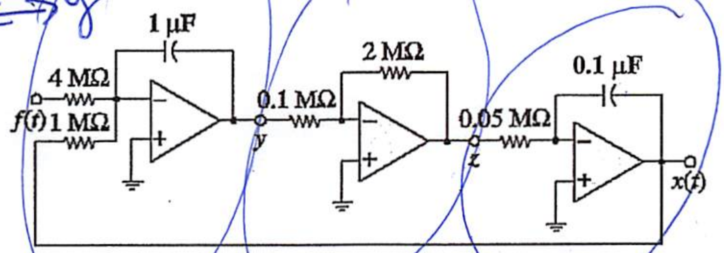


Figure 2

Tech 2

Solution:

$$x(t) = -200\int z(t)\,dt \Rightarrow z(t) = -\frac{1}{200}\frac{dx}{dt}$$

$$z(t) = -20y(t) \Rightarrow y(t) = -\frac{1}{20}z(t) = \frac{1}{4000}\frac{dx}{dt} \qquad ...(1)$$

And

$$y(t) = -\frac{1}{4}\int f(t)\,dt - \int x(t)\,dt \Rightarrow \frac{dy}{dt} = -\frac{1}{4}f(t) - x(t) \qquad ...(2)$$

Combining (1) and (2) gives:

ANSWER

$$\frac{1}{4000}\frac{d^2x}{dt^2} = -\frac{1}{4}f(t) - x(t)$$

$$\boxed{\frac{d^2x}{dt^2} + 4000x(t) = -1000f(t)}$$

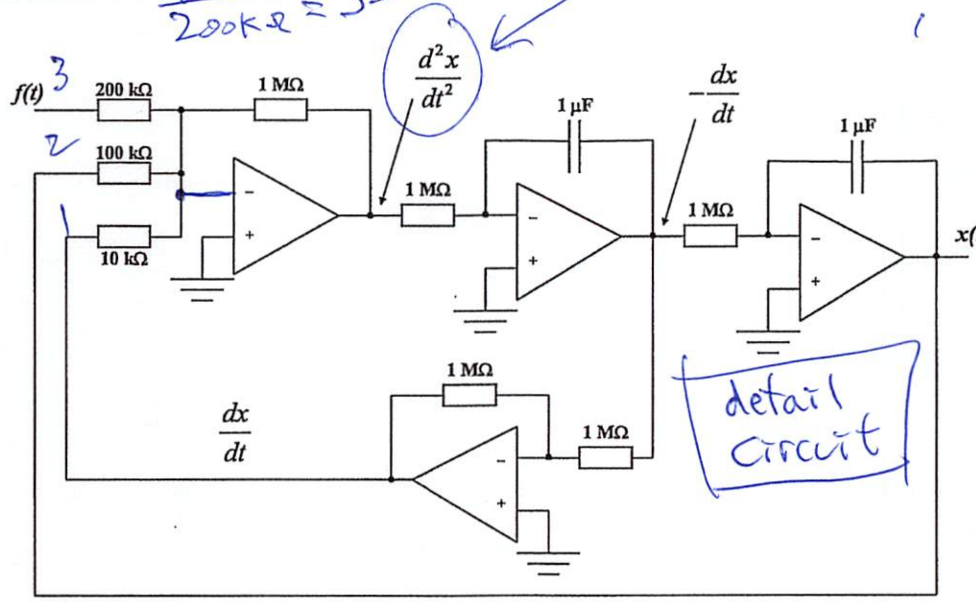
3. Construct the analog computer simulation corresponding to the following differential equation:

$$\frac{d^2x}{dt^2}+100\frac{dx}{dt}+10x=-5f(t)$$

$$\frac{d^2x}{dt} = -100\frac{dx}{dt} - 10x - 5f(t)$$

1 2 3

Solution: $\frac{1\,M\Omega}{200\,k\Omega} = 5$



$\frac{1\,\Omega}{}$ 1F

100kΩ
10µF